

# A systematic study of finite BRST-BV transformations in field-antifield formalism

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## Abstract

We study systematically finite BRST- BV transformations in the field-antifield formalism. We present explicitly their Jacobians and the form of a solution to the compensation equation determining the functional field dependence of finite Fermionic parameters, necessary to generate arbitrary finite change of gauge-fixing functions in the path integral.

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# 1 Introduction

It is well known that BRST symmetry [1, 2, 3] discovered first for non-abelian fields within Faddeev-Popov method [4] is the fundamental principle in modern quantizations of arbitrary gauge systems in both Hamiltonian and Lagrangian formalisms. Parameters of that symmetry are constant Fermions although they are allowed to be functionals of fields. Usually, the symmetry is introduced infinitesimally, which means that its Fermionic parameters are considered formally as infinitely-small quantities. Usual strategy is to show that the Jacobian of BRST transformation does generate arbitrary variation of gauge-fixing functions in the path integral. This can be done by choosing necessary functional dependence of BRST parameters on fields.

The idea to generalize the BRST symmetry for finite Fermionic parameters appears quite natural. Historically, there were several authors (see [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] and references herein) who worked sporadically with finite BRST transformations. But the final results were formulated infinitesimally even in these special cases. In our recent article [16], we have proposed a systematic study of finite BRST (with using abbreviation BRST-BFV) transformations in the generalized Hamiltonian (BFV) formalism [5, 6, 9]. We have developed a unique consistent approach to that matter. Its new strategy was to show that the Jacobian of these finite BRST-BFV transformations does generate arbitrary finite change of gauge-fixing functions in the path integral. In order to do that, we have formulated the so-called compensation equation determining the necessary functional field dependence for finite Fermionic parameters.

In the present paper we will extend our investigations of finite BRST transformations in Lagrangian (covariant) formalism. The covariant quantization of gauge theories has made a long way starting from the famous work of Feynman [17] where S-matrix non-unitarity in Yang-Mills theories and also in Einstein gravity within naive quantization rules has been found. Later the emerged problem was solved by Faddeev and Popov [4], and DeWitt [18]. Many authors have contributed to developing the methods of covariant quantization, as well as to providing them with various applications. More references can be found in books [14, 19]. Discovery of supergravity theories [20, 21, 22] and covariant quantization of antisymmetric tensor fields involved new types of gauge models for which gauge transformations do not form a gauge group [23, 24]. Direct application of the Faddeev-Popov rules [4] leads in the case of these theories to an incorrect result connecting with the non-unitarity of physical  $S$ -matrix. The reason lies in the structure of gauge transformations for these theories. In this case, the arising structure coefficients may depend on the fields of the initial theory, and the gauge algebra of these transformations may be opened by terms proportional to the equations of motion. Moreover, attempts of covariant quantization of gauge theories with linearly-dependent generators of gauge transformations result in the understanding of the fact that it is impossible to use the Faddeev-Popov rules to construct a suitable quantum theory [25, 26]. In turn consistent quantization of supergravity [27, 28] required involving new types of ghosts (known now as

Nielsen-Kallosh ghosts). Therefore, the covariant quantization of gauge theories demanded in general taking into account many new aspects such as open algebras, reducible generators and so on as well as using new approaches. A unique closed approach to the problem of covariant quantization summarized all these features and attempts was proposed by Batalin and Vilkovisky [7, 8]. The Batalin-Vilkovisky (BV) or field-antifield formalism gives the rules for the quantization of general gauge theories. In the field-antifield formalism, there is a number of rather specific features caused by the nontrivial aspects of the antisymplectic geometry [29]. The coexistence/interaction between the odd antibracket and the odd Laplacian should be mentioned first of all in that case.

In the present article, we will develop a unique consistent approach based on the use of BRST symmetry with finite Fermionic parameters, in the framework of the field - antifield formalism [7, 8]. We will refer to these transformations as finite BRST-BV transformations. In principle, our new construction does follow the same general logic as we did in our previous article [16].

Finally, let us note that for Yang-Mills theories within the Faddeev-Popov method [4], an attempt to study of finite BRST transformations was undertaken in [30] where a differential equation for the Jacobian of such change of variables in vacuum functional has been proposed, but a solution to this equation has not been found. Recently [31] it was proved that the problem of finding an explicit form of the Jacobian in Yang-Mills theories is pure algebraic and can be solved in terms of the BRST variation of field-dependent parameter. Any finite BRST transformation of variables in the generating functional of Green functions is related to modification of gauge fixing functional [31, 32].

## 2 Finite BRST-BV Transformations and Their Jacobians

Let

$$z^\alpha = \{\Phi^A; \Phi_A^*\}, \quad (2.1)$$

be a set of Darboux coordinates of field-antifield phase space, whose Grassmann parities are

$$\varepsilon_A = \varepsilon(\Phi^A), \quad \varepsilon(\Phi_A^*) = \varepsilon_A + 1. \quad (2.2)$$

Every anticanonical pair in (2.1) consists of field  $\Phi^A$  and antifield  $\Phi_A^*$ , so that the statistics of antifield is opposite to that of field, in accordance with (2.2). In what follows below, we will mean the set (2.1) in the sense of condensed DeWitt's notations, being the capital indices like  $\{A\}$  the corresponding condensed indices of fields and antifields.

In terms of (2.1), the path integral for the partition function reads

$$Z_\Psi = \int D\Phi D\Phi^* D\lambda \exp \left\{ \frac{i}{\hbar} W_\Psi \right\}, \quad (2.3)$$

where the gauge-fixed quantum master action is defined by

$$W_\Psi = W(\Phi, \Phi^*) + G_A \lambda^A, \quad (2.4)$$

with

$$G_A = \Phi_A^* - \Psi(\Phi) \frac{\overleftarrow{\partial}}{\partial \Phi^A}, \quad (2.5)$$

being just a gauge condition eliminating the antifields in terms of a field dependent gauge-fixing Fermion  $\Psi(\Phi)$ . The dynamical quantum master action  $W(\Phi, \Phi^*)$  is defined by the quantum master equation

$$\Delta \exp \left\{ \frac{i}{\hbar} W \right\} = 0, \quad (2.6)$$

where

$$\Delta = (-1)^{\varepsilon_A} \frac{\partial}{\partial \Phi^A} \frac{\partial}{\partial \Phi_A^*}, \quad (2.7)$$

is a nilpotent odd Laplacian operator

$$\varepsilon(\Delta) = 1, \quad \Delta^2 = \frac{1}{2} [\Delta, \Delta] = 0. \quad (2.8)$$

In (2.3), we have also integrated over the Lagrange multipliers  $\lambda^A$  with Grassmann parity  $\varepsilon(\lambda^A) = \varepsilon_A + 1$ . That integration just generates in (2.3) the gauge-fixing  $\delta$ -function  $\delta(G)$  of (2.5).

The quantum master equation is rewritten in its quadratic form convenient for  $\hbar$ -expansion,

$$\frac{1}{2} (W, W) = i\hbar \Delta W, \quad (2.9)$$

where on the left-hand side we have used the so-called antibracket,

$$(F, G) = F \left( \frac{\overleftarrow{\partial}}{\partial \Phi^A} \frac{\overrightarrow{\partial}}{\partial \Phi_A^*} - \frac{\overleftarrow{\partial}}{\partial \Phi_A^*} \frac{\overrightarrow{\partial}}{\partial \Phi^A} \right) G = -(G, F) (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)}. \quad (2.10)$$

The antibracket (2.10) is odd,

$$\varepsilon((F, G)) = \varepsilon(F) + \varepsilon(G) + 1, \quad (2.11)$$

it satisfies the Leibnitz rule,

$$(F, GH) = (F, G)H + G(F, H) (-1)^{(\varepsilon(F)+1)\varepsilon(G)}, \quad (2.12)$$

it satisfies the Jacobi identity,

$$((F, G), H) (-1)^{(\varepsilon(F)+1)(\varepsilon(H)+1)} + \text{cycle}(F, G, H) = 0, \quad (2.13)$$

and it is differentiated by the  $\Delta$  operator (2.7),

$$\Delta(F, G) = (\Delta F, G) - (F, \Delta G)(-1)^{\varepsilon(F)}. \quad (2.14)$$

There exists also the well-known Witten formula,

$$\Delta(FG) = (\Delta F)G + F(\Delta G)(-1)^{\varepsilon(F)} + (F, G)(-1)^{\varepsilon(F)}, \quad (2.15)$$

where the  $(FG)$  on the left-hand side means the ordinary product, not the antibracket.

In the simplest particular case we have the fundamental antibracket of the anticanonical form,

$$(\Phi^A, \Phi^B) = 0, \quad (\Phi_A^*, \Phi_B^*) = 0, \quad (\Phi^A, \Phi_B^*) = \delta_B^A. \quad (2.16)$$

In terms of the antibracket (2.10), the gauge-fixing functions  $G_A$  do commute among themselves,

$$(G_A, G_B) = 0, \quad (2.17)$$

so that the condition  $G_A = 0$  specifies a Lagrangian hyper-surface in the field-antifield phase space.

Now, let us define finite field dependent BRST-BV transformations<sup>4</sup>,

$$\overline{\Phi}^A = \Phi^A + \lambda^A \mu, \quad (2.18)$$

$$\overline{\Phi}_{*A}^* = \Phi_A^* + \mu \left( W \frac{\overleftarrow{\partial}}{\partial \Phi^A} \right), \quad (2.19)$$

$$\overline{\lambda}^A = \lambda^A, \quad (2.20)$$

$$\mu = \mu(\Phi, \lambda), \quad \varepsilon(\mu) = 1. \quad (2.21)$$

Thus, finite Fermionic parameter  $\mu$  is only allowed to depend on fields  $\Phi^A$  and dynamically-passive Lagrange multipliers  $\lambda^A$ .

It follows from (2.18) - (2.20) that the transformed gauge-fixed quantum master action (2.4) has the form

$$\begin{aligned} \overline{W}_\Psi = W_\Psi|_{z \rightarrow \bar{z}} &= W_\Psi + W \frac{\overleftarrow{\partial}}{\partial \Phi^A} \lambda^A \mu + \mu \left( W \frac{\overleftarrow{\partial}}{\partial \Phi^A} \right) \frac{\partial}{\partial \Phi_A^*} W + \\ &+ \mu \left( W \frac{\overleftarrow{\partial}}{\partial \Phi^A} \right) \lambda^A - \Psi \left( \frac{\overleftarrow{\partial}}{\partial \Phi^A} \lambda^A \right)^2 \mu = W_\Psi - \frac{1}{2}(W, W)\mu. \end{aligned} \quad (2.22)$$

In (2.22), on the right-hand side of the second equality, the second term cancels the fourth one, while the fifth term is zero by itself due to the nilpotency of the operator squared.

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<sup>4</sup> Notice that the transformations (2.18), (2.19) for the fields  $\Phi$  and  $\Phi^*$  are really anticanonical for  $\mu = \text{const}$  and  $\lambda^A$  on the “mass-shell”  $\partial W_\Psi / \partial \Phi_A^* = 0 \Rightarrow \lambda^A = -\partial W / \partial \Phi_A^*$ .

The transformations (2.18), (2.19) cause the following set of elements of the Jacobi matrix,

$$\overline{\Phi}^A \frac{\overleftarrow{\partial}}{\partial \Phi^B} = \delta_B^A + \lambda^A \mu \frac{\overleftarrow{\partial}}{\partial \Phi^B}, \quad (2.23)$$

$$\overline{\Phi}^A \frac{\overleftarrow{\partial}}{\partial \Phi_B^*} = 0, \quad (2.24)$$

$$\overline{\Phi}^*_{\phantom{*}A} \frac{\overleftarrow{\partial}}{\partial \Phi^B} = \left[ \mu \left( W \frac{\overleftarrow{\partial}}{\partial \Phi^A} \right) \right] \frac{\overleftarrow{\partial}}{\partial \Phi^B}, \quad (2.25)$$

$$\overline{\Phi}^*_{\phantom{*}A} \frac{\overleftarrow{\partial}}{\partial \Phi_B^*} = \delta_A^B + \mu \left( W \frac{\overleftarrow{\partial}}{\partial \Phi^A} \frac{\overleftarrow{\partial}}{\partial \Phi_B^*} \right). \quad (2.26)$$

Due to (2.24), the complete Jacobian  $J$  of the transformation of the variables (2.1),

$$J = \text{sDet} \left\{ \bar{z}^\alpha \frac{\overleftarrow{\partial}}{\partial z^\beta} \right\}, \quad (2.27)$$

factorizes to the product of Jacobians of the blocks (2.23) and (2.26). In this way, we have

$$J = J_\Phi J_{\Phi^*}, \quad (2.28)$$

where

$$J_\Phi = \text{sDet} \left\{ \overline{\Phi}^A \frac{\overleftarrow{\partial}}{\partial \Phi^B} \right\} = \text{sDet} \left\{ \delta_B^A + \lambda^A \left( \mu \frac{\overleftarrow{\partial}}{\partial \Phi^B} \right) \right\}, \quad (2.29)$$

$$J_{\Phi^*} = \text{sDet} \left\{ \overline{\Phi}^*_{\phantom{*}A} \frac{\overleftarrow{\partial}}{\partial \Phi_B^*} \right\} = 1 - (\Delta W) \mu. \quad (2.30)$$

It follows from (2.22), (2.30) that

$$\begin{aligned} \exp \left\{ \frac{i}{\hbar} \overline{W}_\Psi \right\} J_{\Phi^*} &= \exp \left\{ \frac{i}{\hbar} W_\Psi \right\} \left[ 1 - \frac{i}{\hbar} \left( \frac{1}{2} (W, W) - i \hbar \Delta W \right) \mu \right] = \\ &= \exp \left\{ \frac{i}{\hbar} W_\Psi \right\}, \end{aligned} \quad (2.31)$$

where we have used the quantum master equation (2.9) in the last equality.

It remains to calculate the factor (2.29). We have

$$\ln J_\Phi = \int_0^1 d\beta G_B^A(\beta) \lambda^B \left( \mu \frac{\overleftarrow{\partial}}{\partial \Phi^A} \right) (-1)^{\epsilon_A}, \quad (2.32)$$

where  $G_B^A(\beta)$  is defined by the equation

$$\left[ \delta_B^A + \beta \lambda^A \left( \mu \frac{\overleftarrow{\partial}}{\partial \Phi^B} \right) \right] G_C^B(\beta) = \delta_C^A. \quad (2.33)$$

It follows immediately from (2.33) that

$$G_B^A = \delta_B^A - \beta \lambda^A (1 + \beta \kappa)^{-1} \left( \mu \frac{\overleftarrow{\partial}}{\partial \Phi^B} \right), \quad (2.34)$$

where the functional  $\kappa$  equals

$$\kappa = \mu \frac{\overleftarrow{\partial}}{\partial \Phi^A} \lambda^A. \quad (2.35)$$

By substituting (2.34) into (2.32) we get the following  $\beta$ -integral

$$\ln J_\Phi = \int_0^1 d\beta [-\kappa + \beta(1 + \beta \kappa)^{-1} \kappa^2] = - \int_0^1 d\beta \kappa (1 + \beta \kappa)^{-1} = - \ln(1 + \kappa), \quad (2.36)$$

so that

$$J_\Phi = (1 + \kappa)^{-1}. \quad (2.37)$$

### 3 Compensation Equation and Its Explicit Solution

Now, we would like to use the field Jacobian (2.37) to generate arbitrary finite change  $\delta\Psi$  of the gauge Fermion  $\Psi$  in the path integral (2.3) with the action (2.4), (2.5),

$$\Psi \rightarrow \Psi_1 = \Psi + \delta\Psi. \quad (3.1)$$

Let us proceed with the path integral in the new variables (2.18) - (2.20),

$$Z_\Psi = \int D\Phi D\Phi^* D\bar{\lambda} \exp \left\{ \frac{i}{\hbar} \overline{W}_\Psi \right\} = \int D\Phi D\Phi^* D\lambda J_\Phi \exp \left\{ \frac{i}{\hbar} W_\Psi \right\}, \quad (3.2)$$

where we have used (2.28), (2.31). In order to provide for the change (3.1), let us require the condition

$$J_\Phi = \exp \left\{ -\frac{i}{\hbar} \left( \delta\Psi \frac{\overleftarrow{\partial}}{\partial \Phi^A} \lambda^A \right) \right\} \quad (3.3)$$

to hold. Then we arrive at

$$Z_{\Psi_1} = Z_\Psi, \quad (3.4)$$

which means the  $\Psi$ -independence for the partition function.

It is also true that the quantum mean value  $\langle \mathcal{O} \rangle_\Psi$  with the weight functional  $\exp\{(i/\hbar)W_\Psi\}$  does not depend on  $\Psi$  for any physical observable  $\mathcal{O}$  annihilated by the Fermionic nilpotent  $\sigma = (W, \dots) - i\hbar\Delta(\dots)$ ,  $\sigma\mathcal{O} = 0$ . The main idea of the proof is that the product  $\mathcal{O} \exp\{(i/\hbar)W\}$  is annihilated by the operator  $\Delta$  due to (2.6) and the definition of  $\sigma$ , so that  $\overline{W} = W + (\hbar/i) \ln \mathcal{O}$  satisfies eq. (2.6) as well. Then we refer to the usual argument with  $\overline{W}$  standing for  $W$ .

Due to (2.35), (2.37), the condition (3.3) is rewritten in the form

$$\mu \frac{\overleftarrow{\partial}}{\partial \Phi^A} \lambda^A = \exp \left\{ \frac{i}{\hbar} \left( \delta \Psi \frac{\overleftarrow{\partial}}{\partial \Phi^A} \lambda^A \right) \right\} - 1. \quad (3.5)$$

We call the condition (3.3) or (3.5) "a compensation equation". Actually, that equation has to determine the necessary field dependence for  $\mu$ .

There exists an obvious explicit solution to the compensation equation (3.5),

$$\mu = \mu(\delta \Psi) = \mu(\Phi, \lambda; \delta \Psi) = \frac{i}{\hbar} f(x) \delta \Psi, \quad (3.6)$$

where the functional  $x$  equals

$$x = \frac{i}{\hbar} \delta \Psi \frac{\overleftarrow{\partial}}{\partial \Phi^A} \lambda^A, \quad (3.7)$$

and

$$f(x) = (\exp(x) - 1) x^{-1}. \quad (3.8)$$

As the operator on the right-hand side in (3.7) is nilpotent, the latter does annihilate the  $x$  (3.7) and any function of it. As the same nilpotent operator stands on the left-hand side in (3.5), it applies nontrivially only to the rightmost factor  $\delta \Psi$  in (3.6), which results exactly in having the factor  $x^{-1}$  in (3.8) canceled. In this way, we have confirmed immediately the equation (3.5) to hold.

Notice, that for finite change  $\delta \Psi$  the solution (3.6) is in general  $\lambda$  dependent. However, in the first order in  $\delta \Psi$ , explicit solution (3.6) takes the usual form

$$\mu(\delta \Psi) = \frac{i}{\hbar} \delta \Psi + O((\delta \Psi)^2). \quad (3.9)$$

which is  $\lambda$  independent as far as the  $\delta \Psi$  does the same.

## 4 $\lambda$ -Differential as a BRST-BV Generator for Fields

Let us consider the Fermionic nilpotent operator

$$\overleftarrow{d} = \frac{\overleftarrow{\partial}}{\partial \Phi^A} \lambda^A, \quad (4.1)$$

$$\epsilon(\overleftarrow{d}) = 1, \quad (\overleftarrow{d})^2 = \frac{1}{2} [\overleftarrow{d}, \overleftarrow{d}] = 0. \quad (4.2)$$

In terms of (4.1) the action (2.4), (2.5) is rewritten as

$$W_\Psi = W(\Phi, \Phi^*) + (\Phi_A^* \Phi^A - \Psi(\Phi)) \overleftarrow{d}. \quad (4.3)$$



The transformation (2.18) of fields  $\Phi^A$  takes the form

$$\overline{\Phi}^A = \Phi^A(1 + \overleftarrow{d}\mu). \quad (4.4)$$

Thus, the operator (4.1) is a generator for finite BRST-BV field transformation.

The formula (2.37) for the Jacobian  $J_\Phi$  is rewritten as

$$J_\Phi = J_\Phi(\mu) = [1 + (\mu \overleftarrow{d})]^{-1}. \quad (4.5)$$

The compensation equation (3.5) takes the form

$$\mu \overleftarrow{d} = \exp \left\{ \frac{i}{\hbar} \left( \delta \Psi \overleftarrow{d} \right) \right\} - 1. \quad (4.6)$$

The  $x$  in (3.7) can be represented as

$$x = \frac{i}{\hbar} \delta \Psi \overleftarrow{d}. \quad (4.7)$$

Thus, we conclude that all the main objects in our consideration can be expressed naturally in terms of a single quantity that is the BRST-BV field generator (4.1), also called "a  $\lambda$ -differential".

Notice that the introduced field transformations (4.4) form a group. Indeed, let us rewrite (4.4) in the form

$$\overline{\Phi}^A = \Phi^A \overleftarrow{T}(\mu), \quad \overleftarrow{T}(\mu) = 1 + \overleftarrow{d}\mu. \quad (4.8)$$

Then the group composition law of the transformations (4.8) reads

$$\overleftarrow{T}(\mu_1) \overleftarrow{T}(\mu_2) = \overleftarrow{T}(\mu_{12}), \quad (4.9)$$

where

$$\mu_{12} = \mu_1 + [J_\phi(\mu_1)]^{-1} \mu_2, \quad (4.10)$$

where  $J_\phi(\mu_1)$  is the Jacobian (4.5) with  $\mu_1$  standing for  $\mu$ . Indeed, due to the nilpotency (4.2) of  $\overleftarrow{d}$ , we have

$$\overleftarrow{T}(\mu_1) \overleftarrow{T}(\mu_2) = 1 + \overleftarrow{d}\mu_1 + \overleftarrow{d}\mu_2 + \overleftarrow{d}\mu_1 \overleftarrow{d}\mu_2 = 1 + \overleftarrow{d}\mu_1 + \overleftarrow{d}\mu_2 + \overleftarrow{d}(\mu_1 \overleftarrow{d})\mu_2. \quad (4.11)$$

By inserting here

$$(\mu_1 \overleftarrow{d}) = [J_\phi(\mu_1)]^{-1} - 1. \quad (4.12)$$

we arrive at (4.10).

Moreover, it follows from (4.9) that the algebra of the group generators has the form

$$[\overleftarrow{d}\mu_1, \overleftarrow{d}\mu_2] = \overleftarrow{d}\mu_{[12]}, \quad (4.13)$$

where

$$\mu_{[12]} = \mu_{12} - \mu_{21} = -(\mu_1 \mu_2) \overleftarrow{d}. \quad (4.14)$$

## 5 Ward Identities Dependent of Finite BRST-BV Parameters/Functionals

As we have defined finite BRST-BV transformations, it appears quite natural to use them immediately to derive the corresponding modified version of the Ward identity. We will do that just in terms of BRST-BV field generator introduced in Section 4.

As usual for that matter, let us proceed with the external-source dependent generating functional

$$Z_\Psi(\zeta) = \int D\Phi D\Phi^* D\lambda \exp \left\{ \frac{i}{\hbar} W_\Psi(\zeta) \right\}, \quad (5.1)$$

where

$$W_\Psi(\zeta) = W_\Psi + \zeta_A \Phi^A, \quad (5.2)$$

$\zeta_A$  are arbitrary external sources to the fields  $\Phi^A$ ,  $\varepsilon(\zeta_A) = \varepsilon_A$ . Notice that we do not introduce their own sources to antifields  $\Phi_A^*$ . Of course, in the presence of non-zero external source, the path integral (5.1) by itself is in general actually dependent of gauge Fermion  $\Psi$ . However, this dependence has a special form and the equivalence theorem [34], applying in physical sector, makes possible to establish that the physical quantities do not depend on gauge. In its turn, the Ward identity measures the deviation of the path integral from being gauge-independent.

Let us perform in (5.1) the change (2.18)-(2.20) of integration variables, with arbitrary finite  $\mu(\phi, \lambda)$ . Then, by using (2.31) and (4.5), we get what we call "a modified Ward identity",

$$\left\langle \left[ 1 + \frac{i}{\hbar} \zeta_A (\Phi^A \overleftarrow{d}) \mu \right] [1 + (\mu \overleftarrow{d})]^{-1} \right\rangle_{\Psi, \zeta} = 1, \quad (5.3)$$

where we have denoted the source dependent mean value

$$\langle (...) \rangle_{\Psi, \zeta} = [Z_\Psi(\zeta)]^{-1} \int D\Phi D\Phi^* D\lambda (...) \exp \left\{ \frac{i}{\hbar} W_\Psi(\zeta) \right\}, \quad \langle 1 \rangle_{\Psi, \zeta} = 1, \quad (5.4)$$

related to the source dependent action (5.2). By construction, in (5.3), both  $\zeta_A$  and  $\mu(\Phi, \lambda)$  are arbitrary. The presence of arbitrary  $\mu(\Phi, \lambda)$  in the integrand in (5.3) reveals the implicit dependence of generating functional (5.1) on the gauge-fixing Fermion  $\Psi$  for non-zero external source  $\zeta_A$ .

For a constant  $\mu$ ,  $\mu = \text{const}$ , the latter does drop-out completely, and we get from (5.3)

$$\langle \zeta_A \lambda^A \rangle_{\Psi, \zeta} = 0. \quad (5.5)$$

By using the representation

$$\lambda^A \exp \left\{ \frac{i}{\hbar} G_B \lambda^B \right\} = \frac{\hbar}{i} \frac{\partial}{\partial \Phi_A^*} \exp \left\{ \frac{i}{\hbar} G_B \lambda^B \right\}, \quad (5.6)$$

and integrating over the antifields  $\Phi_A^*$  by part, (5.5) is rewritten as

$$\left\langle \zeta_A \left( \frac{\partial}{\partial \Phi_A^*} W \right) \right\rangle_{\Psi, \zeta} = 0, \quad (5.7)$$

which is exactly the standard form of a Ward identity in the field-antifield formalism.

By identifying the  $\mu$  in (5.3) with the solution (3.6) to the compensation equation (4.6), it follows according to our result in Section 3,

$$Z_{\Psi_1}(\zeta) = Z_{\Psi}(\zeta) \left[ 1 + \left\langle \frac{i}{\hbar} \zeta_A (\Phi^A \overleftarrow{d}) \mu(-\delta\Psi) \right\rangle_{\Psi, \zeta} \right]. \quad (5.8)$$

Formula (5.8) generalizes the gauge independence (3.4) of the partition function to the presence of the external source.

## 6 Discussions

We have introduced the conception of finite BRST-BV transformations in the field-antifield quantization formalism [7, 8] for general gauge-field dynamical systems. It was shown that the Jacobian of finite BRST-BV transformations, being the main ingredient of the approach, can be calculated explicitly in terms of the corresponding generator applied to finite field-dependent functional parameters of these transformations. We have introduced the compensation equation providing for a connection between the generating functionals formulated for a given dynamical system in two different gauges. We have extended the proof of gauge independence of partition function and quantum mean values of physical observables as to the case of finite variations of gauge-fixing functional. We have found an explicit solution to the compensation equation proposed. We have studied the algebra and the group composition law of finite BRST-BV transformations. As a by-product, we have developed a technique using the so-called  $\lambda$ -differential, provided for deriving in a simple way the Ward identity and connection between the generating functionals of Green functions written in two different gauges.

In conclusion, we would like to present in short an alternative view on the role of finite BRST transformations as respected by the path integral (2.3). Let us rewrite the latter in the form

$$Z_{\Psi} = \int D\Phi D\Phi^* D\lambda \exp \left\{ \frac{i}{\hbar} (W + X) \right\}, \quad (6.1)$$

where  $W$  satisfies (2.6) while  $X$  is given by

$$X = G_A \lambda^A \quad (6.2)$$

with  $G_A$  defined in (2.5). Due to (2.17), it follows that

$$\Delta \exp \left\{ \frac{i}{\hbar} X \right\} = 0, \quad (6.3)$$

which is symmetric to (2.6). In the integrand in (6.1), we have the BRST-BV symmetry as represented in its infinitesimal form

$$\delta z^\alpha = (z^\alpha, -W + X)\mu + \frac{\hbar}{i}(z^\alpha, \mu). \quad (6.4)$$

Of course, in principle, we could try to reformulate (6.4) at the level of finite transformations, similarly to what we did with respect to (2.18), (2.19). Instead of doing that here, let us consider now finite equivalence transformations acting on the space of solutions to (6.3),

$$\exp\left\{\frac{i}{\hbar}X'\right\} = \exp\{-[F, \Delta]\} \exp\left\{\frac{i}{\hbar}X\right\}, \quad (6.5)$$

where a function  $F$  is a finite Fermionic generator. Due to the relation

$$[F, \Delta] = (\Delta F) - (F, \dots), \quad (6.6)$$

we have

$$X' = \exp\{(F, \dots)\}X + i\hbar f((F, \dots))\Delta F, \quad (6.7)$$

where  $(F, \dots)$  means the left adjoint action of the antibracket,

$$(F, \dots)G = (F, G), \quad (6.8)$$

and the function  $f(x)$  is given by (3.8). On the right-hand side in (6.7), the first term is an anti-canonical transformation with finite Fermionic generator  $F$ , while the second term is a half of a logarithm of the Jacobian of that transformation up to  $(-i\hbar)$ . If we choose

$$F = -\delta\Psi(\Phi) \quad (6.9)$$

with finite  $\delta\Psi$ , then

$$X' = G'_A \lambda^A, \quad (6.10)$$

where

$$G'_A = \Phi_A^* - \Psi' \frac{\overleftarrow{\partial}}{\partial \Phi^A}, \quad (6.11)$$

and

$$\Psi' = \Psi + \delta\Psi. \quad (6.12)$$

One can rewrite the formula (6.7) as

$$X' - X = -f((F, \dots))\sigma_X F, \quad (6.13)$$

where

$$\sigma_X = -i\hbar \exp \left\{ -\frac{i}{\hbar} \right\} \Delta \exp \left\{ \frac{i}{\hbar} \right\} = (X, \dots) - i\hbar \Delta, \quad (6.14)$$

while in the second equality we have used the quantum master equation (6.3).

An infinitesimal form of (6.3)

$$\delta X = -\sigma_X F + O(F^2), \quad (6.15)$$

shows that  $-\sigma_X$  is a generator to the corresponding variation  $\delta X$ . As the operator  $\Delta$  is nilpotent, it follows from the first equality in (6.14) that the operator  $\sigma_X$  is nilpotent as well.

Of course, it should be noticed that the equivalence transformation (6.7) as presented above, was not generated by any change of integration variables in (6.1). However it can be shown that such a change of variables can be deduced from (6.4) by "integrating" the latter to the level of finite transformation.

In the present article, we have explored the " $Sp(1)$ -version" of the BRST symmetry with a single Fermionic parameter, in the field-antifield formalism. Few years ago, we have proposed the  $Sp(2)$ -version of the field-antifield formalism, based on the conception of the extended BRST symmetry with two Fermionic parameters [35, 36, 37]. It seems very interesting to extend the results obtained above to the case of  $Sp(2)$ -version of the BRST symmetry.

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